DAY 1:

MARA NORD PROJECT - TRAINING 2010

LESSON 2: ELECTROMAGNETIC THEORY, PRINCIPLES OF WAVE PROPAGATION

INTERREG IV A NORD

Statens vegvesen

TRAFIKVERKET

OULUN SEUDUN AMMATTIKORKEAKOULU

Liikennevirasto

Rovaniemen ammattikorkeakoulu University of Applied Sciences

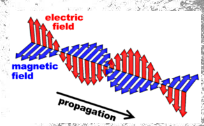
Centre for Economic Development, Transport and the Environment

The theoretical basis to GPR

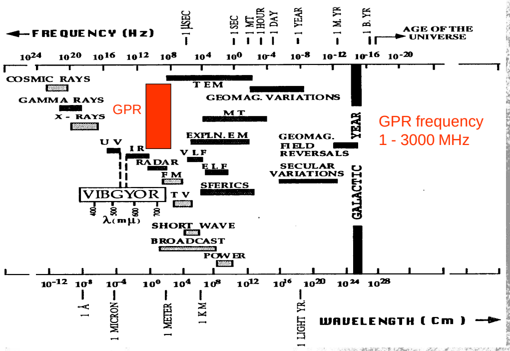
- Issues:
- Electromagnetic waves (EM-waves)
- Basic concepts
- Properties of media affecting the propagation
- Wave propagation: signal attenuation, reflection, and refraction
- Resolution, horizontal and vertical
- Electrical properties of geological materials
- Radar equation

Electromagnetic Waves

- EM radiation fills the universe
- EM wave propagation velocity in vacuum is the same as for light (300 000 km/s)
- EM waves consists of altering electrical and magnetic fields



Electromagnetic spectrum



THE ELECTROMAGNETIC SPECTRUM

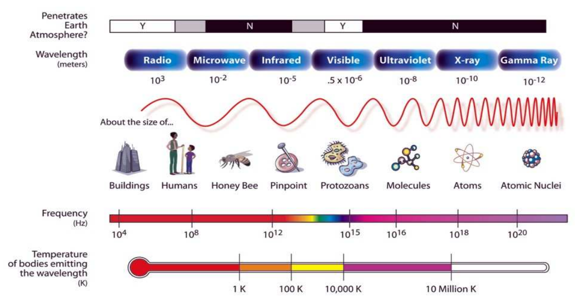
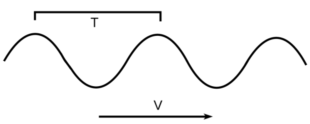


Image by NASA

Basic concepts and Units

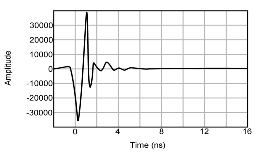
- Time unit t 1 ns = 10^{-9} s = 0.000 000 001 s
- Frequency f 1 MHz = 10^{6} Hz = 1000 000 Hz ($^{1}/_{s}$)
- Period ´T 'peak to peak ($^{1}/_{f}$)
- Velocity v m/ns (m/μs) = v = λf
- Velocity of light c = 0.3 m/ns ≈ 299 792 458 m/s
- Wave length λ m λ = v/f
- Decibel 20 x log(A1/A2) dB (comparison of amplitudes)



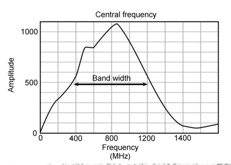
Time and frequency level

- Measurement data can be presented either as a function of time (time domain) or as a function of frequency (frequency domain)
- Conversion between time and frequency domain using Fourier transform. (Based on assumption that any signal can be described as a sum of sinus waves of specified frequency, amplitude and phase)
- Observing the frequency content of the signal

Amplitude plotted against time



Amplitude spectrum plotted against frequency



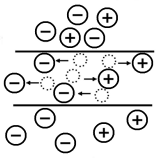
3/25/2011 Material prepared for Mara Nord Project by Roadscanners Oy 7

Electrical properties of the medium (1)

The electrical properties of the medium affect the propagation and behaviour of EM waves in the medium.

Influencing parameters:

Electric conductivity σ [S/m] (apparent resistivity ρ = 1/ σ [Ωm]) influence through electric current density J=σE, which reflects the free movement of charges (ions, electrons) in media caused by an external electrical field (diffusive (non-recurrent), energy loss)



No electrical field

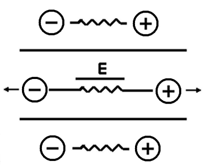
Influence of electrical field

Electrical field removed

3/25/2011 Material prepared for Mara Nord Project by Roadscanners Oy 8

Electrical properties of the medium (2)

- Control the behaviour and propagation of EM waves in the media
- Dielectric value $\varepsilon = \varepsilon_r \varepsilon_0$ (also K):
- Influence through the electric flux density D=εE, which reflects the movement of ions, charges, or polar molecules between two extremes, caused by an external electric field (recurrent)
- Dielectricity is usually presented as a complex value: $\varepsilon = \varepsilon' + j\,\varepsilon''$, where ε' reflects the capacity of the material to store energy, imaginary part ε'' reflects disturbances in charging



No electric field

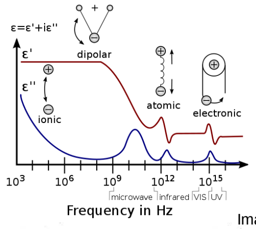
Influence of electric field E

Electric field removed

3/25/2011 Material prepared for Mara Nord Project by Roadscanners Oy 9

Electrical properties of the medium (3)

- There are several modes of polarization which influence differently in different frequencies
- The most significant polarization mode for GPR measurements is the molecular (dipolar) polarization - and the most significant molecule is water molecule H_2O



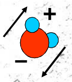


Image by Dr. Kenneth A. Mauritz

Electrical properties of the medium (4)

- Magnetic permeability $\mu = \mu_r \mu_0$ Influence the behaviour of EM waves and their propagation in the medium :
- Influence through magnetic flux density B= μH, which reflects the ability of the material to get magnetized by an external magnetic field
- μ_r value for natural materials is usually low, exceptions are magnetite and other minerals which contain Fe^{3+}
- The influence is usually neglected

EM wave propagation

$$\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \partial \mathbf{D}/\partial t$$

Wave Equation

$$\nabla^2 E = j\omega\mu\sigma E - \omega^2 \mu\epsilon E$$

Propagation Equation

$$E(z,t) = E_o\, e^{-\alpha z}\, e^{j(\omega t - \beta z)}$$

Attenuation coefficient

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right)}$$ (attenuation)

Phase coefficient

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right)}$$ (velocity)

Relation $\sigma/\epsilon\omega$ defines wave propagation in the medium → Radar frequency area 1 - 3000 MHz

Attenuation of the EM wave

- **When the wave can propagate ==>**
- **Simplified formulas for attenuation coefficient and propagation velocity :**

$$\alpha = \frac{1635{,}5 \cdot \sigma}{\sqrt{\varepsilon_r}} \qquad v = \frac{c}{\sqrt{\varepsilon_r}} \qquad \lambda = \frac{v}{f} = \frac{c}{f \cdot \sqrt{\varepsilon_r}}$$

attenuation **velocity** **wave length**

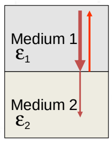
$$A(L) = A(L_0)\,\frac{e^{(-\alpha(L-L_0))}}{(L/L_0)^n}$$

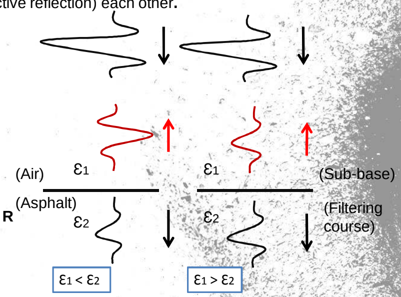
amplitude at distance L

$A(L)$ field param. (E, H) at distance L
$A(L_0)$ field param. (E, H) at zero point
α attenuation coefficient
n 0 1-dimensional (tube wave)
1/2 2-dimensional directed wave (e.g. inside ice)
1 circular wave
2 interface wave

Reflection of the EM wave

- EM wave reflects from an interface, where the electrical properties of the medium change. If the layers are thin, the reflections coming from several layer interfaces may strengthen (constructive reflection) or weaken (destructive reflection) each other.





Reflection coefficient R

$$R = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$

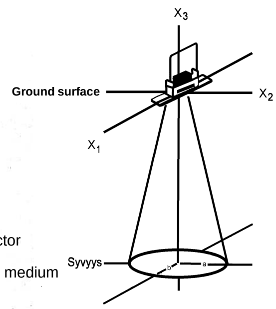
Reflection of the EM wave

- The reflection area (footprint):
- The area, from which the reflections strengthen each other (1st Fresnel zone). The media properties in the area define the strength of reflections.

$$A = \sqrt{\frac{\lambda L}{2}}$$

$$B = \frac{A}{2}$$

L = Depth to reflector
λ = wavelength in medium

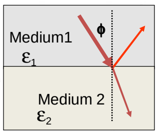


400 MHz, Er=7, L=1m → sqr(0,28m/2)=0,37m

Refraction of the EM waves

- **When a radar signal meets an interface in an angle**
- **ϕ ≠ 0°**
- A part of the signal is reflected,
- Arriving angle = reflection angle
- A part of the signal refracts through the interface

$$\sin\phi_1 = \sin\phi_0$$



Typical reflection contrasts and corresponding reflection coefficients :

From:	Er	To:	Er	R	(dB)
Air	1	Dry soil	5	-0,38	-8,4
Soil	5	Wet soil	25	-0,38	-8,4
Soil	5	Bedrock	8	-0,12	-19
Soil	25	Bedrock	8	0,28	-11
Water	81	Mud	50	0,12	-18
Water	81	Bedrock	8	0,52	-5,7
Ice	3,2	Water	81	-0,67	-3,5
Soil	8	Wet soil	25	-0,34	-9,3
	3-50	Metal	(indef.)	-1	0

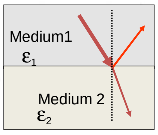
$$v_2 \cdot \sin\phi_1 = v_1 \cdot \sin\phi_2$$

$$\sqrt{\varepsilon_1} \cdot \sin\phi_1 = \sqrt{\varepsilon_2} \cdot \sin\phi_2$$

3/25/2011 Material prepared for Mara Nord Project by Roadscanners Oy 16

Refraction of the EM waves

- If $\varepsilon_1 < \varepsilon_2$ (typical case for GPR) EM waves refract towards normal of the interface → waves focus
- If $\varepsilon_1 > \varepsilon_2$ all EM wave energy reflects completely at the boundary if the angle is greater than the critical angle
- (ϕ_2 = 90 degrees)



$$\phi_1 = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

- Typically ε increases downwards → reflected wave usually meets total reflection. This limits the visible area.

For ε = 6 to air → 24 degrees

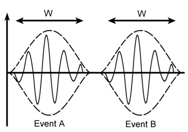
$$v_2 \cdot \sin\phi_1 = v_1 \cdot \sin\phi_2$$

$$\sqrt{\varepsilon_1} \cdot \sin\phi_1 = \sqrt{\varepsilon_2} \cdot \sin\phi_2$$

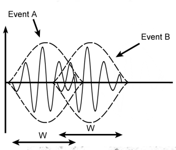
3/25/2011 Material prepared for Mara Nord Project by Roadscanners Oy 17

Depth or Range Resolution of the Measurements

- Range Resolution defines, how close two layer interfaces can be, until radar can no longer resolve them as two separate layers.



Distinct reflections



Reflections get mixed

$$h = \frac{\tau \cdot c}{2 \cdot \sqrt{\varepsilon_r}}$$

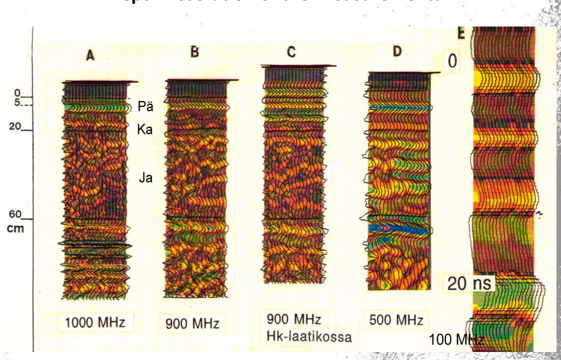
τ pulse width in ns
ε_r relative dielectricity of the media
c light speed 0.3 m/ns

For 1000 MHz (1ns) Er=6
→ 6,1cm

For 400 MHz (2,5ns) Er=7
→ 14cm

3/25/2011 Material prepared for Mara Nord Project by Roadscanners Oy 18

Depth Resolution of the Measurements



Range Resolution of the Measurements

Central frequency (MHz)	Band width (MHz)	Pulse width (ns)	Resolution (m)
200.0	200.0	5	0.25
100.0	100.0	10	0.50
50.0	50.0	20	1.00
25.0	25.0	40	2.00
12.5	12.5	80	4.00

Range Resolution versus dielectric constant for common GPR bandwidths

Another formula for defining the range resolution (Steven Koppenjan (GPR Book, 2009) cites, Eaves and Reedy, 1987)

$$R_{res} = \frac{1.39c}{2B\sqrt{\varepsilon_r}}$$

For 1000 MHz, Er=6 → 8,5cm

For 400 MHz, Er=7 → 20cm

C = speed of light (m/s)
B = bandwidth (Hz)
Er = dieletric permittivity
1.39 = empirical factor

INTERREG IV A NORD

Range Resolution versus dielectric constant for common GPR bandwidths

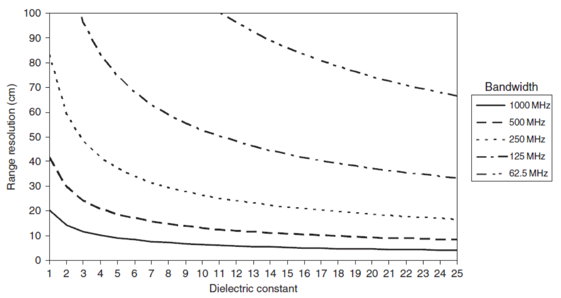


Image by Steven Koppenjan

INTERREG IV A NORD

Horizontal resolution of GPR measurements

- Describes how close in horizontal dimension two point reflectors can be to still distinguish them as two separate objects
- Describes in addition, how steep inclining layers are observed
- Definition in meter / scan maximum distance between points

$$\Delta x = \frac{75}{f \cdot \sqrt{\varepsilon_r}}$$

f frequency (MHz)
e.g. $\varepsilon_r = 9$
100 MHz ⇒ 0.25 m
500 MHz ⇒ 0.05 m !
e.g. $\varepsilon_r = 7$
500 MHz ⇒ 0.06 m

If the number of scans is sufficient, horizontal resolution for point reflectors:

$$\Delta x = 3.3 \cdot \sqrt{\frac{d}{\alpha}}$$

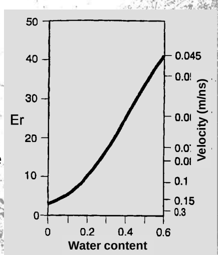
d object depth (m)
α attenuation (dB/m)

INTERREG IV A NORD

Geological materials

- The electromagnetic properties of geological materials are mainly defined by the material's
- volumetric water content K = 81
- air content K = 1
- solids content K = 4 - 5

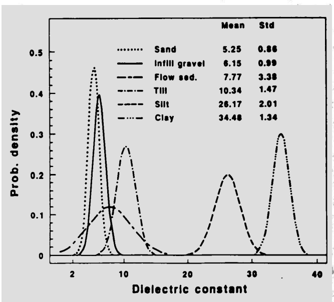
- The values are also influenced by:
- the structure of the medium, the size and shape
- of the structural particles
- the electrochemical characteristics of the interface
- temperature, pressure, density,
- mineral composition etc.



INTERREG IV A NORD

Geological materials

- Electromagnetic properties of geological materials



3/25/2011 Material prepared for Mara Nord Project by Roadscanners Oy 25

INTERREG IV A NORD

Geological materials

Definition of the electromagnetic properties of geological materials :

- Back calculation from reference values
- WARR, CDP, and RSAD sounding
- Hyperbola information
- Measuring dielectrical values with TDR or Percometer
- Modelling

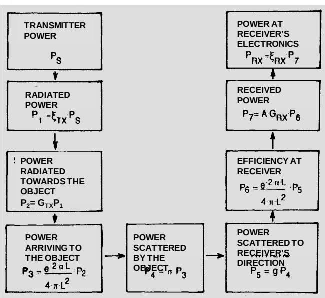
These are described in Interpretation section

3/25/2011 Material prepared for Mara Nord Project by Roadscanners Oy 26

INTERREG IV A NORD

Radar equation

- Depth penetration of the GPR signal: radar equation



ξ_{TX} = Efficiency of the transmitter
G_{TX} = Signal gaining at the transmitter
L = Distance from the object (same as the receiver's)
α = Attenuation coefficient
σ = Scattering area of the object
g = Back scattering gain
ξ_{RX} = Efficiency of the receiver
G_{RX} = Signal gaining at receiver
A = Effective area of the receiver antenna $c^2/4\pi f^2$
c = in the medium
f = Signal frequency

3/25/2011 Material prepared for Mara Nord Project by Roadscanners Oy 27